ValsCalc

Chapter 9 Multiple-Choice Practice Test

Directions: This practice test features multiple-choice questions based on the content in Chapter 9: Parametric Equations and Polar Coordinates.

9.1: Parametric Equations
9.2: Differentiating and Integrating Parametric Functions
9.3: Polar Coordinates and Functions
9.4: Differentiating Polar Functions
9.5: Areas with Polar Curves
9.6: Additional Calculus with Parametric and Polar

For each question, select the best answer provided and do your figuring in the margins. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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Parametric Equations and Polar Coordinates

Number of Questions—50

NO CALCULATOR

1. A point given in polar coordinates by $(6, 300^\circ)$ is expressed in Cartesian coordinates by

(A) $\left(-3, 3\sqrt{3}\right)$

(B) $\left(3, 3\sqrt{3}\right)$

(C) $\left(3, -3\sqrt{3}\right)$

(D) $\left(3\sqrt{3}, 3\right)$

(E) $\left(3\sqrt{3}, -3\right)$

2. The polar equation for the line $y = 12$ is

(A) $r = 12$

(B) $r = 12\cos\theta$

(C) $r = 12\sin\theta$

(D) $r = 12\sec\theta$

(E) $r = 12\csc\theta$

3. The polar curve $r = \cos 5\theta$ is

(A) a limacon with one loop

(B) a circle of radius 1

(C) a circle of radius 5

(D) a rose with five petals

(E) a rose with 10 petals

4. If $\omega \neq 0$, then which set of equations parameterizes the ellipse $\frac{x^2}{64} + \frac{y^2}{81} = 1$?

(A) $x = 8\sin\omega t, \quad y = 8\sin\omega t$

(B) $x = 8\sin\omega t, \quad y = 8\cos\omega t$

(C) $x = 8\sin\omega t, \quad y = 9\cos\omega t$

(D) $x = 9\sin\omega t, \quad y = 8\sin\omega t$

(E) $x = 9\sin\omega t, \quad y = 9\cos\omega t$

5. For the polar curve $r = \cos^3\theta$, $\frac{dx}{d\theta} =$

(A) $-4\sin\theta\cos^2\theta$

(B) $-4\sin\theta\cos^3\theta$

(C) $4\sin\theta\cos^3\theta$

(D) $\cos^4\theta - 3\sin^2\theta\cos^2\theta$

(E) $\cos^4\theta + 3\sin^2\theta\cos^2\theta$

6. If $x = t^2 + 4$ and $y = 3t^3 - 6t$, then $\dfrac{dy}{dx} =$

(A) $9t^2 - 6$ (B) $\dfrac{9t^2 - 6}{2t}$ (C) $\dfrac{2t}{9t^2 - 6}$ (D) $\dfrac{3t^3 - 6t}{t^2 + 4}$ (E) $\dfrac{t^2 + 4}{3t^3 - 6t}$

7. The line $y = 4x$ is represented in polar coordinates by

(A) $r = 4$

(B) $r = \tan^{-1} 4$

(C) $r = 4\theta$

(D) $\theta = 4$

(E) $\theta = \tan^{-1} 4$

8. If $r = \cot\theta$, then $\dfrac{d^2y}{d\theta^2} =$

(A) $-\sin\theta$

(B) $-\cos\theta$

(C) $\sin\theta$

(D) $\cos\theta$

(E) $2\csc^2\theta\cot\theta$

9. If $x = \cos 2t$ and $y = t - 6$, then

(A) $x = \cos(y+6)$

(B) $x = \cos(y-6)$

(C) $x = \cos(y-12)$

(D) $x = \cos(2y+12)$

(E) $x = \cos(2y-12)$

10. If $f(\theta)=1+2\cos\theta$, then which statement is true about the polar curve $r=f(\theta)$?

(A) Since $f\left(\frac{\pi}{4}\right)>0$ and $f'\left(\frac{\pi}{4}\right)<0$, the graph is moving toward the pole when $\theta=\frac{\pi}{4}$.

(B) Since $f\left(\frac{\pi}{4}\right)>0$ and $f'\left(\frac{\pi}{4}\right)<0$, the graph is moving away from the pole when $\theta=\frac{\pi}{4}$.

(C) Since $f\left(\frac{\pi}{4}\right)>0$ and $f'\left(\frac{\pi}{4}\right)>0$, the graph is moving toward the pole when $\theta=\frac{\pi}{4}$.

(D) Since $f\left(\frac{\pi}{4}\right)<0$ and $f'\left(\frac{\pi}{4}\right)<0$, the graph is moving away from the pole when $\theta=\frac{\pi}{4}$.

(E) Since $f\left(\frac{\pi}{4}\right)<0$ and $f'\left(\frac{\pi}{4}\right)>0$, the graph is moving away from the pole when $\theta=\frac{\pi}{4}$.

11. On a platform 5 feet above the ground, a ball is thrown upward at an angle of 30° above the horizontal with a speed of 40 feet per second. If x is the ball's horizontal position from the platform and y is its height above the ground, then which set of parametric equations represents the ball's motion?

(A) $x = 20t$, $\quad y = -16t^2 + 20t\sqrt{3} + 5$

(B) $x = 20t$, $\quad y = -16t^2 - 20t\sqrt{3} + 5$

(C) $x = 20t\sqrt{3}$, $\quad y = -16t^2 - 20t + 5$

(D) $x = 20t\sqrt{3}$, $\quad y = -16t^2 + 20t + 5$

(E) $x = 40t$, $\quad y = -16t^2 + 40t + 5$

12. Which limacon has an inner loop?

(A) $r = 2 - \sin\theta$

(B) $r = 1 - \cos\theta$

(C) $r = 6 + 6\cos\theta$

(D) $r = 8 + 4\sin\theta$

(E) $r = 3 - 8\cos\theta$

13. If $x = 3t + 7$ and $y = \sqrt{t^3 - t - 5}$, then

(A) $y = \sqrt{\frac{x^3}{27} - \frac{x}{3} - 5}$

(B) $y = \sqrt{(x-7)^3 - (x-7) - 5}$

(C) $y = \sqrt{(3x+7)^3 - (3x+7) - 5}$

(D) $y = \sqrt{\frac{(x+7)^3}{27} - \frac{x+7}{3} - 5}$

(E) $y = \sqrt{\frac{(x-7)^3}{27} - \frac{x-7}{3} - 5}$

14. For the polar curve $r = 3 + \cos\theta$, $\dfrac{dy}{dx} =$

(A) $\dfrac{-3\cos\theta - \cos 2\theta}{3\sin\theta + \sin 2\theta}$

(B) $\dfrac{-3\cos\theta - \cos 2\theta}{3\cos\theta + \cos^2\theta}$

(C) $\dfrac{3\cos\theta + \cos^2\theta}{3\sin\theta + \sin\theta\cos\theta}$

(D) $\dfrac{3\sin\theta + \sin 2\theta}{-3\cos\theta - \cos 2\theta}$

(E) $\dfrac{3\sin\theta + \sin\theta\cos\theta}{3\cos\theta + \cos^2\theta}$

15. A curve is parameterized by the equations $x = \sqrt{t+3}$ and $y = 3t^2 - 8t + 2$. When $x = 2$, $\dfrac{dy}{dx} =$

(A) -8 (B) $-\dfrac{3}{2}$ (C) $-\dfrac{2}{\sqrt{5}}$ (D) 4 (E) $8\sqrt{5}$

16. What is the length of the curve parameterized by $x = \frac{2}{3}t^{3/2} + 8$ and $y = 2t^{3/2} - 4$ on the interval $0 \leqslant t \leqslant 2$?

(A) $\dfrac{2}{3}\sqrt{10}$ (B) $\dfrac{2}{3}\sqrt{80}$ (C) $\dfrac{8}{3}\sqrt{8}$ (D) $\dfrac{8}{3}\sqrt{10}$ (E) $\dfrac{8}{3}\sqrt{80}$

17. Which forms of symmetry does the polar curve $r = 2 - 3\sin\theta$ exhibit?

I. Symmetry about the x-axis

II. Symmetry about the y-axis

III. Symmetry about the origin

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

18. The area enclosed by $r = 2 + \cos\theta$ is

(A) 2π (B) 4π (C) $\dfrac{9\pi}{4}$ (D) $\dfrac{9\pi}{2}$ (E) 9π

19. Which equation represents the tangent line to the curve parameterized by $x = \sqrt[3]{t}$ and $y = t^3 - 4t^2 + 8$ at $t = 1$?

(A) $y - 5 = -15(x - 1)$

(B) $y - 5 = -5(x - 1)$

(C) $y - 5 = -\dfrac{1}{15}(x - 1)$

(D) $y - 5 = \dfrac{1}{3}(x - 1)$

(E) $y - 5 = 5(x - 1)$

20. A particle travels counterclockwise along a circle of radius 4 centered at $(5, -2)$. The particle begins at the top of the circle and completes one revolution over $0 \leqslant t \leqslant 3$. Which parametric equations describe the particle's motion?

(A) $x = 5 + 4\cos\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\sin\left(\frac{2\pi}{3}t\right)$

(B) $x = 5 - 4\cos\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\sin\left(\frac{2\pi}{3}t\right)$

(C) $x = 5 + 4\sin\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\cos\left(\frac{2\pi}{3}t\right)$

(D) $x = 5 - 4\sin\left(\frac{2\pi}{3}t\right), \qquad y = -2 + 4\cos\left(\frac{2\pi}{3}t\right)$

(E) $x = 5 - 4\sin\left(\frac{2\pi}{3}t\right), \qquad y = -2 - 4\cos\left(\frac{2\pi}{3}t\right)$

21. An equation of the line tangent to the polar curve $r = 4\sin\theta$ at $\theta = \dfrac{\pi}{6}$ is

(A) $y - 1 = 2\left(x - \sqrt{3}\right)$

(B) $y - 1 = \sqrt{3}\left(x - \sqrt{3}\right)$

(C) $y - 1 = \dfrac{1}{\sqrt{3}}\left(x - \sqrt{3}\right)$

(D) $y - \sqrt{3} = \sqrt{3}\,(x - 1)$

(E) $y - \sqrt{3} = \dfrac{1}{\sqrt{3}}\,(x - 1)$

22. When $\theta = \dfrac{\pi}{3}$, the distance between the polar curves $r = 5\cos\theta$ and $r = 1 + 2\cos\theta$ is

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) 2 (D) $\dfrac{5}{2}$ (E) $\dfrac{3\sqrt{3}}{2}$

23. The area of the region enclosed by the curve $x = \sin t$, $y = 2t$ for $0 \leqslant t \leqslant \pi$ is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) 8

24. If $x = 3\cos 2t$ and $y = 2\sin 2t$, then $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} =$

(A) $-\frac{2}{9}\csc^3 2t$

(B) $-\frac{2}{3}\cot 2t$

(C) $\frac{2}{3}\tan 2t$

(D) $\frac{4}{3}\csc^2 2t$

(E) $\frac{1}{3}\csc^2 2t \sec 2t$

25. The arc length of the curve parameterized by $x = 6t + 5$ and $y = 8 - \frac{16}{3}t^{3/2}$ for $0 \leqslant t \leqslant 1$ is

(A) $\dfrac{1}{16}$ (B) 4 (C) $\dfrac{49}{6}$ (D) 10 (E) $\dfrac{1568}{3}$

26. The area of the region that is outside the cardioid $r = 4 - 4\cos\theta$ and also inside the circle $r = 4$ is

(A) 4π (B) 8π (C) $16 - 2\pi$ (D) $32 - 4\pi$ (E) $64 - 8\pi$

27. If $x = 6 - e^{-8t}$ and $\dfrac{d^5y}{dx^5} = 3\ln t$, then $\dfrac{d^6y}{dx^6} =$

(A) $\dfrac{3}{8te^{-8t}}$ (B) $\dfrac{3}{6t - te^{-8t}}$ (C) $\dfrac{3\ln t}{6 - e^{-8t}}$ (D) $\dfrac{8te^{-8t}}{3}$ (E) $\dfrac{6 - e^{-8t}}{3\ln t}$

28. The area enclosed by $r^2 = 8\cos 2\theta$ is

(A) $\sqrt{2}$ (B) $\sqrt{8}$ (C) 2 (D) 4 (E) 8

29. A cycloid has a peak at $(10\pi, 20)$. Parametric equations for the cycloid are

(A) $x = 10(1 - \cos\theta), \qquad y = 10(\theta - \sin\theta)$

(B) $x = 10(\theta - \sin\theta), \qquad y = 10(1 - \cos\theta)$

(C) $x = 20(1 - \cos\theta), \qquad y = 20(1 - \sin\theta)$

(D) $x = 20(1 - \cos\theta), \qquad y = 20(\theta - \sin\theta)$

(E) $x = 20(\theta - \sin\theta), \qquad y = 20(1 - \cos\theta)$

30. The graph parameterized by $x = 6 - 4t^2$ and $y = t^4 - 2$ is concave up for

(A) $t < 0$

(B) $t < \frac{\sqrt{6}}{2}$

(C) $t > 0$

(D) $t > \sqrt[4]{2}$

(E) $-\infty < t < \infty$

31. Which integral equals the length of the polar curve $r = \cos^2\theta$ over $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$?

(A) $\displaystyle\int_0^{\pi/2} \sqrt{1+4\sin^2\theta\cos^2\theta}\,\mathrm{d}\theta$

(B) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^2\theta+2\sin\theta\cos\theta}\,\mathrm{d}\theta$

(C) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^2\theta-2\sin\theta\cos\theta}\,\mathrm{d}\theta$

(D) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^4\theta+4\sin^2\theta\cos^2\theta}\,\mathrm{d}\theta$

(E) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^4\theta-4\sin^2\theta\cos^2\theta}\,\mathrm{d}\theta$

32. Half of a petal on the curve $r = \cos 2\theta$ is revolved around the x-axis to generate a solid whose surface area is given by

(A) $2\pi \int_0^{\pi/4} \sin\theta \cos 2\theta \sqrt{1+4\sin^2 2\theta}\, d\theta$

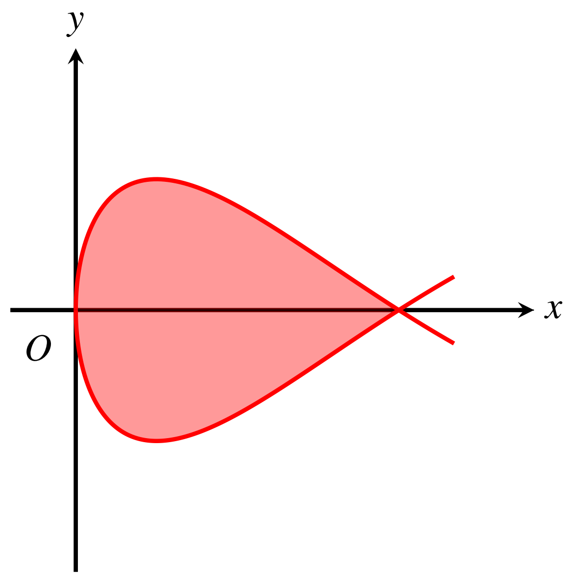
(B) $2\pi \int_0^{\pi/4} \sin\theta \cos 2\theta \sqrt{\cos^2 2\theta - 4\sin^2 2\theta}\, d\theta$

(C) $2\pi \int_0^{\pi/4} \sin\theta \cos 2\theta \sqrt{\cos^2 2\theta + 4\sin^2 2\theta}\, d\theta$

(D) $2\pi \int_0^{\pi/4} \cos\theta \cos 2\theta \sqrt{\cos^2 2\theta - 4\sin^2 2\theta}\, d\theta$

(E) $2\pi \int_0^{\pi/4} \cos\theta \cos 2\theta \sqrt{\cos^2 2\theta + 4\sin^2 2\theta}\, d\theta$

33. The graph parameterized by $x = t^2$ and $y = \sin 2t$ traces out a teardrop, as shown below.



The area of the teardrop is

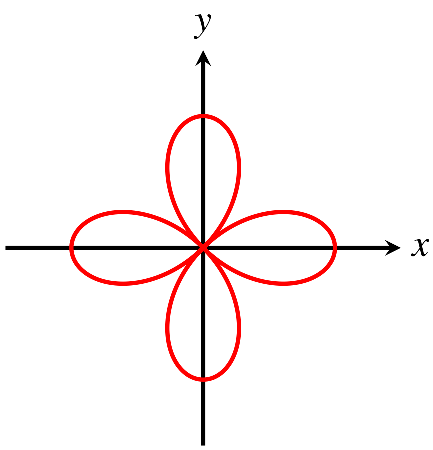
(A) $\dfrac{\pi}{8}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{2}$ (D) π (E) 2π

34. The area of the region enclosed by $r = 4\cos\theta$ and $r = 4\sin\theta$ is

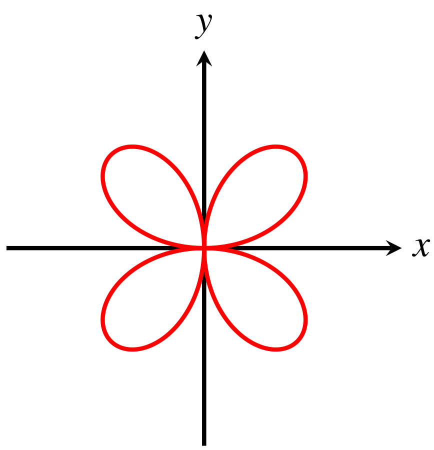
(A) 4 (B) $\pi + 4$ (C) $2\pi - 2$ (D) $2\pi - 4$ (E) $2\pi + 4$

35. Which choice is the graph of $r = \sin 4\theta$?

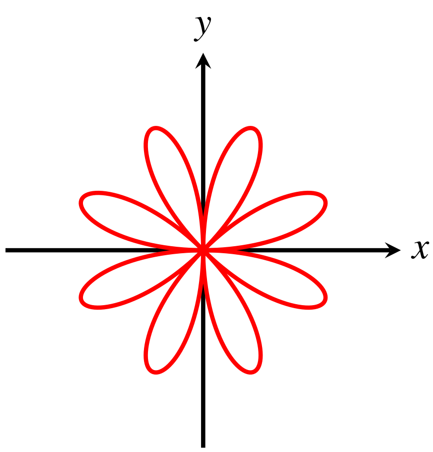
(A)



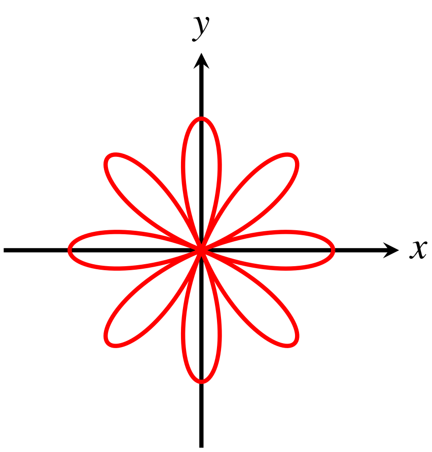
(B)



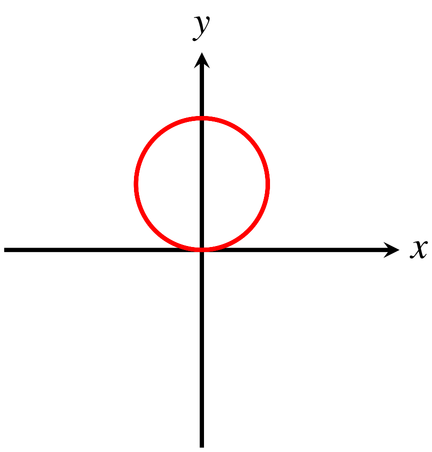
(C)



(D)



(E)



36. The area enclosed by the inner loop of the limacon $r = 2 - 4\cos\theta$ is

(A) $2\pi + 3\sqrt{3}$

(B) $2\pi - 3\sqrt{3}$

(C) $4\pi + 6\sqrt{3}$

(D) $4\pi - 6\sqrt{3}$

(E) $8\pi - 12\sqrt{3}$

37. The graph of $\left(x^2 + y^2\right)^3 = 4x^2y^2$ is identical to which polar curve?

(A) $r = \sin\theta$

(B) $r = \sin 2\theta$

(C) $r = \cos 2\theta$

(D) $r = 2\sin 2\theta$

(E) $r = 2\cos 2\theta$

38. The area of the region bounded by $r = 4\theta$, where $0 \leqslant \theta \leqslant \pi$, and the x-axis is

(A) $\frac{\pi^2}{2}$ (B) π^2 (C) $\frac{4\pi^3}{3}$ (D) $\frac{8\pi^3}{3}$ (E) $\frac{16\pi^3}{3}$

39. For $0 \leqslant \theta \leqslant \frac{\pi}{2}$, the tangent to the polar curve $r = 4\sin\theta$ is 1 when

(A) $\theta = \frac{\pi}{16}$ (B) $\theta = \frac{\pi}{8}$ (C) $\theta = \frac{\pi}{6}$ (D) $\theta = \frac{\pi}{4}$ (E) $\theta = \frac{\pi}{2}$

40. Curve C is parameterized by $x = 5 + 3\cos t$ and $y = 3\sin t$ for $0 \leqslant t \leqslant \frac{\pi}{2}$. The surface area of revolution in rotating C about the x-axis is

(A) $6\pi\sqrt{18}$ (B) $12\pi\sqrt{18}$ (C) 9π (D) 18π (E) $15\pi^2$

41. For $0 \leqslant \theta \leqslant \frac{\pi}{2}$, the polar curve $r = 9\cos\theta$ has a horizontal tangent when

(A) $\theta = 0$ (B) $\theta = \frac{\pi}{6}$ (C) $\theta = \frac{\pi}{4}$ (D) $\theta = \frac{\pi}{3}$ (E) $\theta = \frac{\pi}{2}$

42. When $\theta = \dfrac{3\pi}{4}$, the polar curve $r = 9\cos 3\theta$ is getting

(A) closer to the x-axis and closer to the y-axis

(B) closer to the x-axis and farther away from the y-axis

(C) farther away from the x-axis and closer to the y-axis

(D) farther away from the x-axis and farther away from the y-axis

(E) farther away from the x-axis, and neither closer to nor farther away from the y-axis

43. The area of *one* petal of the rose $r = \cos 8\theta$ is

(A) $\dfrac{\pi}{64}$ (B) $\dfrac{\pi}{32}$ (C) $\dfrac{\pi}{16}$ (D) $\dfrac{\pi}{8}$ (E) $\dfrac{\pi}{4}$

44. A woman standing on a balcony 10 meters above the ground throws a flying disk upward with a speed of 20 meters per second at an angle of 45° to the horizontal. If x is the disk's horizontal distance from the woman and y is its height above the ground, then the disk's trajectory is modeled by

(A) $y = 10 + x + \dfrac{9.8x^2}{200}$

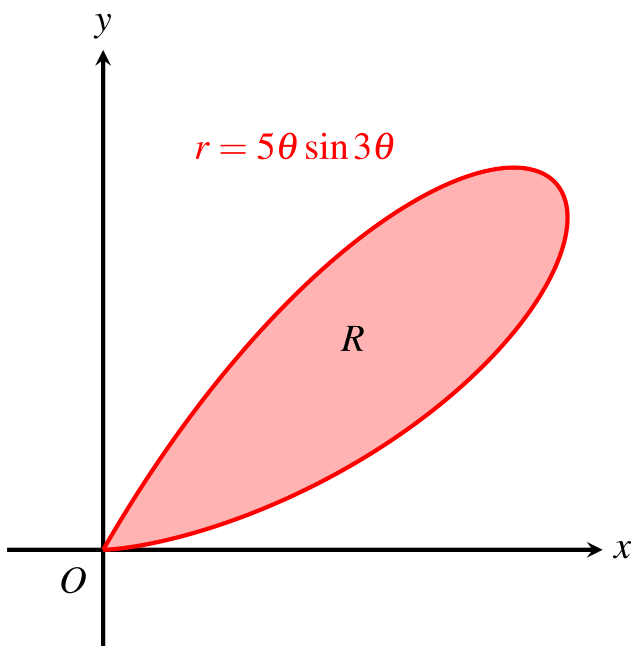
(B) $y = 10 + x + \dfrac{9.8x^2}{400}$

(C) $y = 10 + x + \dfrac{9.8x^2}{800}$

(D) $y = 10 + x - \dfrac{9.8x^2}{400}$

(E) $y = 10 + x - \dfrac{9.8x^2}{800}$

Questions 45 and 46 refer to the following region.



45. Which integral equals the area of R ?

(A) $\frac{1}{2}\int_{-\pi/2}^{0} 25\theta^2 \sin^2 3\theta \, d\theta$

(B) $\frac{1}{2}\int_{0}^{\pi/6} 25\theta^2 \sin^2 3\theta \, d\theta$

(C) $\frac{1}{2}\int_{0}^{\pi/3} 25\theta^2 \sin^2 3\theta \, d\theta$

(D) $\frac{1}{2}\int_{0}^{\pi/2} 25\theta^2 \sin^2 3\theta \, d\theta$

(E) $\frac{1}{2}\int_{0}^{\pi} 25\theta^2 \sin^2 3\theta \, d\theta$

46. The average distance from O to any point on the boundary of R is

(A) $\frac{5}{9}$ (B) $\frac{5}{6}$ (C) $\frac{5}{3}$ (D) $\frac{5\pi}{9}$ (E) $\frac{5\pi}{3}$

47. The region S is bounded by the line $\theta = \frac{\pi}{2}$ and the polar curve $r = 7\theta$. Which integral equals the surface area of the solid generated by rotating S about the line $\theta = \frac{\pi}{2}$?

(A) $49\pi \int_0^{\pi/2} \theta \sin\theta \sqrt{\theta^2+1}\, d\theta$

(B) $49\pi \int_0^{\pi/2} \theta \cos\theta \sqrt{\theta^2+1}\, d\theta$

(C) $98\pi \int_0^{\pi/2} \sin\theta \sqrt{\theta^2+1}\, d\theta$

(D) $98\pi \int_0^{\pi/2} \theta \sin\theta \sqrt{\theta^2+1}\, d\theta$

(E) $98\pi \int_0^{\pi/2} \theta \cos\theta \sqrt{\theta^2+1}\, d\theta$

48. Which expression equals the area of the region that is inside $r = 6 - 4\cos\theta$ and also inside $r = 4$?

(A) $\int_0^{\pi} (2 - 4\cos\theta)^2 \, d\theta$

(B) $\int_0^{\pi/3} 16 \, d\theta + \int_{\pi/3}^{\pi} (6 - 4\cos\theta)^2 \, d\theta$

(C) $\int_0^{\pi/3} (6 - 4\cos\theta)^2 \, d\theta + \int_{\pi/3}^{\pi} 16 \, d\theta$

(D) $\int_0^{\pi} \left[(6 - 4\cos\theta)^2 - 16\right] \, d\theta$

(E) $\int_0^{\pi} \left[16 - (6 - 4\cos\theta)^2\right] \, d\theta$

49. If C is the curve parameterized by $x = \sin t$ and $y = e^t$ for $0 \leqslant t \leqslant 1$, then the area between C and the x-axis is

(A) $\dfrac{e\sin 1 + e\cos 1 - 1}{2}$

(B) $\dfrac{1 + e\sin 1 + e\cos 1}{2}$

(C) $\dfrac{1 + e\sin 1 - e\cos 1}{2}$

(D) $\dfrac{1 - e\sin 1 + e\cos 1}{2}$

(E) $\dfrac{1 - e\sin 1 - e\cos 1}{2}$

50. Let $A(k)$ be the area that is enclosed by the polar curves $r = 6\cos\theta$ and $r = k\sin\theta$, where k is a positive constant. Then $\lim\limits_{k\to\infty} A(k)$ is

(A) $\dfrac{9\pi}{2}$ (B) 9π (C) 18π (D) 36π (E) nonexistent

This marks the end of the test. The following page contains the answers to all the questions.

1. C
2. E
3. D
4. C
5. B
6. B
7. E
8. B
9. D
10. A
11. D
12. E
13. E
14. A
15. A
16. B
17. B
18. D
19. A
20. D
21. B
22. B
23. D
24. A
25. C
26. D
27. A
28. E
29. B
30. E
31. D
32. C
33. D
34. D
35. C
36. D
37. B
38. D
39. B
40. D
41. C
42. A
43. B
44. D
45. C
46. C
47. E
48. C
49. A
50. A